Developing A Learning Trajectory for Teaching Statistics

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Abstract: This paper aimed to develop a learning trajectory (LT) for teaching statistics using Realistic Mathematics Education (RME) approach and investigating its influence on students’ reasoning ability. The LT was developed through a design research that consisted of a cyclic process of preparing for the experiment, conducting the experiment, and retrospective analysis. The research’s subjects were 30 students at grade 9 MTsN Salido, Indonesia. Data were collected through observations, interviews, checklist, videotaping, and analyzing the students’ works. The LT was validated by three experts in mathematics education and one expert in instructional design and it met the criteria of validity (relevance and consistency). The LT was evaluated through one-to-one and small group evaluation before it was tried out in a field test. The results showed that the LT worked as intended in the classroom. It also could help the students to reinvent statistical concepts such as mode, mean, median, quartile, and deviation standard. Finally, we discovered the growth in the students’ statistical reasoning.

Keywords: Learning trajectory; RME; HLT; statistical reasoning.

1. Introduction

The needs to develop a learning trajectory (LT) for teaching a certain mathematics topic are increased nowadays. That is because researchers and mathematics educators realize that LTs play very important roles in building students’ understanding of mathematical concepts. Gravemeijer [1] mentioned that if we want students to reinvent mathematics by doing mathematics, teachers have to adapt to how their students reason and help them build on their own thinking. To do so, teachers need to design a framework of reference (LT). LTs are very helpful for bridging the work of researchers and practitioners [2]. LTs also can help teachers evaluating and rethink teaching, which enable them to have a general vision of the class before they start teaching [3,4].

Many LTs in mathematics were developed by researchers. They were not only for teaching mathematics in primary and secondary education (see e.g. [5-8]) but also for teaching certain courses in higher education (see e.g. [9-11]). In general, the results of the researches revealed that the LTs were very helpful in building student’s conceptual understanding.

In this research we developed a LT for teaching statistics using RME approach for the students at grade 9 in junior high school because of two main reasons. Firstly, statistics is a very important topic in mathematics that has many applications in daily life and in the other subjects [12]. Secondly, mathematics textbooks and teachers tend to present statistics topic in mechanistic way in which the formulas were introduces in the first places, followed by giving some examples that show how the formulas works [13].

To focus the research, we formulated three research questions. Firstly, what are the characteristics of a valid learning trajectory for teaching statistics using RME approach? Secondly, Firstly, what are the characteristics of a practical learning trajectory for teaching statistics using RME approach?
Thirdly, to what extend the learning trajectory for teaching statistics using RME approach could stimulate students’ reasoning?

2. Literature Review

In this section we discuss the results of literature review about LT, hypothetical learning trajectory, and RME. A LT is the sequences of activities and tasks that might support the development of students’ understanding of a specific instructional goal [14]. Gravemeijer [15] said that neither teachers, nor researchers can rely on fixed teaching sequences, since a teacher continuously has to adapt to the actual thinking and learning of her students. Therefore, the preliminary version of a LT is in form of a hypothetical one, and it is called a hypothetical learning trajectory (HLT) [14-16].

A HLT consists of three components: the learning goal that defines the direction, the learning activities, and the hypothetical learning processes—a prediction and anticipation of how the students’ thinking and understanding will evolve in the context of the learning activities [14]. After a cyclic process of designing, testing, and re-designing, a HLT becomes a theory (LT) that can be used as a lesson learned by other mathematics educators to teach a certain mathematics topic. Gravemeijer [15] and Liljekvist [16] called the theory as a local instructional theory (LIT), while Cobb et al [17] called it as domain specific theories. A local instruction theory consists of theories about both the process of learning a specific topic and the means to support that learning [1].

We employed RME approach in this research because the idea of developing a LT is in line with the idea of how mathematics has to be taught in RME. Process of learning mathematics in RME can be described as a phenomenon of an iceberg in Figure 1 [11].

![Figure 1. Learning Mathematics in RME as a Phenomenon of an Iceberg](image)

A very strong foundation is needed to support the top of the iceberg to appear on sea surface. In relation to this phenomenon, formal and abstract mathematical concepts are situated on the top of the iceberg. Mathematics educators or researchers need to provide a strong foundation and ‘a best trajectory’ for students to reach the top of the iceberg. To do so, at the beginning of the lesson, students are provided with contextual problems that can be solved using their informal knowledge. The contextual problems will also facilitate students to use their own symbols or their own strategy. This process is called horizontal mathematization. After experiencing similar processes and empowering by simplification and formalization (see [18]), students will use more formal language or strategies in solving contextual problems. The journey, that will bring students to re-invent a formal mathematical, is called vertical mathematization [15, 19 20].

The LT designed in this research was based on three key principles of RME for instructional designed, namely guided reinvention trough progressive mathematization, didactical phenomenology, and emerging models [15,20]. Meanwhile, the implementation of the LT in the classrooms was guided by RME’s characteristics [18, 21-23].
3. Method

This research used design research approach proposed by Gravemeijer and Cobb [24]. We used the approach because design research aims at understanding more of the interrelatedness between teaching and learning in order to improve teaching [1]. Design research in this study consisted of a cyclic process of preparing for the experiment, conducting the experiment, and retrospective analysis. Gravemeijer and Cobb illustrated the cyclic process as can be seen in Figure 2.

![Figure 2. The Cyclic Process of Design Research](image)

In preparing for the experiment we determined the end points of the instructions. The goals of our statistics lessons were the students reinvent the concepts of mode, mean, median, quartile, and deviation standard. As the students were already familiar with data in daily life, we used data created by the students as the starting points of the lessons. After we set the end and the starting points, we designed the HLT that consisted of four main activities and ten sub-activities of solving contextual problems that would facilitate students to do horizontal and vertical mathematization as well as stimulate students’ thinking and reasoning. In addition, we also formulated the predictions of students’ thinking and solutions, and the anticipations.

In the experiment phase, we tried out the HLT in three cycles. The first try out was conducted in one-to one setting that involved three students. The second try out was conducted in small group that involved six students at grade 9 MTsN Salido, Indonesia. After the retrospective analysis and re-design processes, the HLT was tried out to 30 students at grade nine in the same school. The retrospective analysis involved the research team, a teacher, and an observer. Besides focusing our attention to develop the HLT, we also observed and analyzed the impact of the HLT on the development of students’ confidence in using their own strategies when solving the contextual problems and the development of students’ reasoning during the try out. Data of the research were collected through observations, interviews, checklist, videotaping, and analyzing the students’ works.

4. Results and Discussion

In the preparing for the experiment, we analyzed the essential concepts about statistics in the curriculum, the literatures about teaching statistics [12], and the characteristics of the students such as their hobbies, their preferences in learning, and their preferences activities. Based on the analysis, we designed the HLT for teaching statistics. The HLT was started by exploring the concept of mode using the context of favorite candies. The students reinvent the concept of mode as the most favorite candy chosen by themselves in the classroom (see Figure 3).
Figure 3. The Students Reinvent the Concept of Mode

The concept was explored further by observing the highest frequency in several diagrams such as a bar, a line, and a pie diagram. One example of the students’ works can be seen in Figure 4.

Figure 4. The Students Explored the Concept of Mode Using Diagrams

The concept of mean was explored by the students by predicting the number of pigeons in a figure using estimation. The students later on related their estimations to the concepts of mean. The other concepts were reinvent by the students using similar learning experiences (exploring contextual problems).

The HLT for teaching statistics was validated by three mathematic education experts in Indonesia during preparing for the experiment phase. The result showed that the HLT met the criteria of validity [25], with the characteristics; the activities of solving contextual problems in the HLT were potential to facilitate the students to reinvent the concepts in statistics; the activities were well sequenced, the HLT suit the key principles and characteristics of RME; and the components in the HLT were well designed and consistent between one and another.

The LT also satisfied the criteria of practicality [25], in which it worked as intended during the try out. The students understood the contextual problems and they conducted ‘doing math’ activities without major obstacle. The probing questions that were prepared as the anticipations of students’ thinking and solutions also helped the students to achieved the goals of the activities. In addition, the time provided for doing the activities of solving contextual problems was well planned. This findings were in line with the previous findings which showed that LT and RME gave positive impact on students’ reasoning ability [26, 27].

5. Conclusion

The LT for teaching statistics using RME approach developed in this research met the criteria of validity, practicality, and effectiveness. Design research approach that we used to develop the LT was very helpful in reaching our goal. The LT for teaching statistics reflected the state of the art of RME and it worked as intended in the classroom. Moreover, the LT could help the students to reinvent the concepts in statistics topic. The students had more confidence to use their own strategies in solving...
contextual problems. The most important thing, we discovered the growth in the students’ statistical reasoning.

References

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